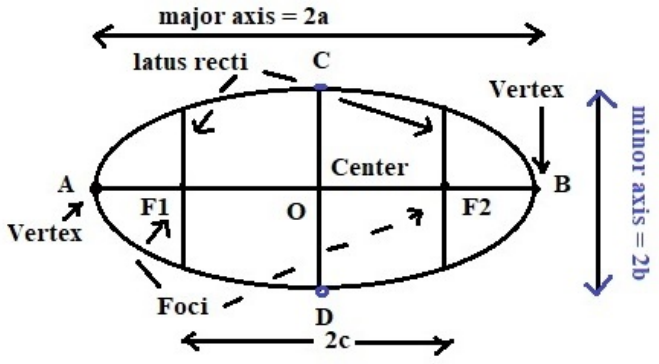
Conic Sections: Ellipse

A **conic section** is the intersection of a right circular cone and a plane parallel to an element of the cone. By changing the angle and the site where the plane slices the cone, you get circles, ellipses, parabolas and hyperbolae.



An **ellipse** is the locus of all points the sum of whose distances from two fixed points, called the **foci**, is a constant. The line segment through the foci of an ellipse is called the **major axis**. The intersections of the ellipse and the major axis are called the **vertices**. Halfway between the foci is the **center** of the ellipse. The line segment passing through the centre of the ellipse perpendicular to the major axis is the **minor axis**. The intersections of the ellipse and the minor axis are called the **co-vertices**. The two **latus recti** pass through the foci, perpendicular to the major axis. The length of each **latus rectum** is equal to $\mathbf{2b^2/a}$.

The **standard equations of an ellipse** are $\mathbf{(x-h)^2/a^2 + (y-k)^2/b^2 = 1}$ and $\mathbf{(x-p)^2/b^2 + (y-q)^2/a^2 = 1}$. The **centers** of these two types of ellipses are **(h, k)** and **(p, q)**, respectively. The distance between the foci is **2c**. Lengths of major and minor axes are **2a** and **2b**, respectively. a and b are also called semi-major axis and semi-minor axis, respectively.

c is **squareroot of** $\mathbf{(a^2 - b^2)}$. $\mathbf{(x-h)^2/a^2 + (y-k)^2/b^2 = 1}$ has a **horizontal major axis** and **vertical minor axis**. $\mathbf{(x-p)^2/b^2 + (y-q)^2/a^2 = 1}$ has a **vertical major axis** and a **horizontal minor axis**.

Eccentricity e of an ellipse describes how "**elongated**" it is. As **e** approaches 0, the shape becomes more circular. As it approaches 1, the ellipse becomes more elongated. e is **c/a**. In the 18th century, Halley had shown that the comet (known as Halley's comet) moved in an elliptical orbit around the sun. Its eccentricity is 0.9675.

Elliptical arches, having two centers and continually changing radii, are found in **arches of doorways** or on **water bridges**. In brick masonry, they are considered one of the strongest arches.

A physical property of ellipses is that **rays** (light or sound) **emanating from one focus** will **reflect back to the other focus**. This has been exploited in medicine as **extracorporeal shock wave lithotripsy**. The patient is placed next to a shock wave-generating machine so that the **kidney/gall stones** are at one focus of the ellipse. The machine emits waves from the other focus, which scatter into the elliptical bowl, and bounce back to reconcentrate at the

other focus inside the patient. As they reach full power at the site of the kidney/gall stones, they smash the stones into small pieces and “sand” that can be rid of by taking an oral bile acid pill. The patient can go home the same day, having had no invasive surgery.