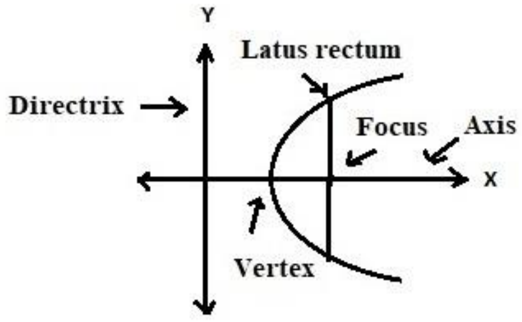
Conic Sections: Parabola

A **conic section** is the intersection of a right circular cone and a plane parallel to an element of the cone. By changing the angle and the site where the plane slices the cone, you get circles, ellipses, parabolas and hyperbolae.



A **parabola** is the locus of all points equidistant from a line called the **directrix**, and a given point, not on the line, called the **focus**. Its **axis of symmetry** passes through the **focus** and **vertex**, and is perpendicular to the **directrix**. The **vertex** is halfway between the **focus** and **directrix**. For **regular** parabolas, the axes of symmetry are parallel to the **y axis** and for "**sideways parabolas**", the axes of symmetry are parallel to the **x axis**. The line segment through the focus of a parabola, perpendicular to the axis of symmetry, is called the **latus rectum**.

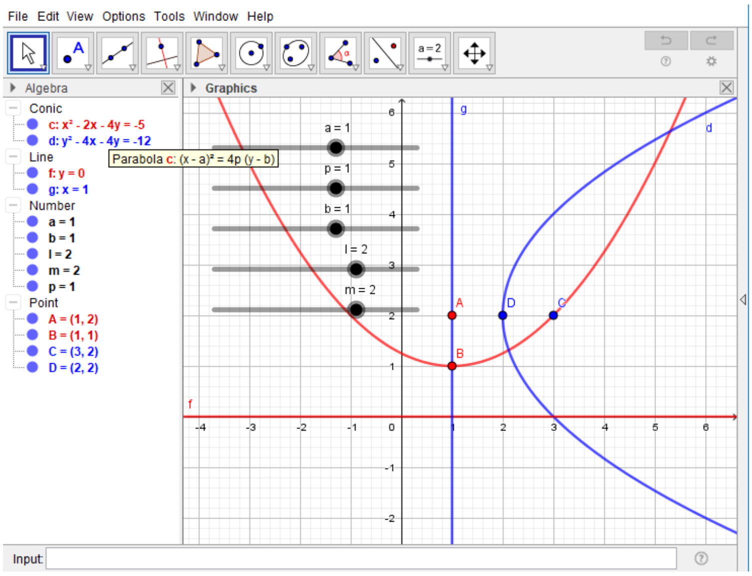
The **general forms** of the equations describing parabolas are $y = ax^2+bx+c$ and $x = ay^2+by+c$ (for sideways parabolas). The **conics forms** are $(x-a)^2 = 4p\ (y-b)$ and $(y-l)^2 = 4p\ (x-m)$ (for sideways parabolas). The **vertices** in these two equations are **(a, b)** and **(l, m),** respectively. The **foci** for parabolas described by these two equations are **(0, p)** and **(p, 0)**, respectively. Length of the **latus rectum** is **4p**.

One **important property** of parabolas has been exploited for many applications such as **bionic ears**, **satellite** and **radar dishes** and **reflectors on torches and spotlights**. This property is that <u>any ray parallel to the axis of symmetry is reflected off the inner surface straight to the focus</u>. That's why this point is called the "**<u>focus</u>**". Thus, the signal is **concentrated** onto a receiver.

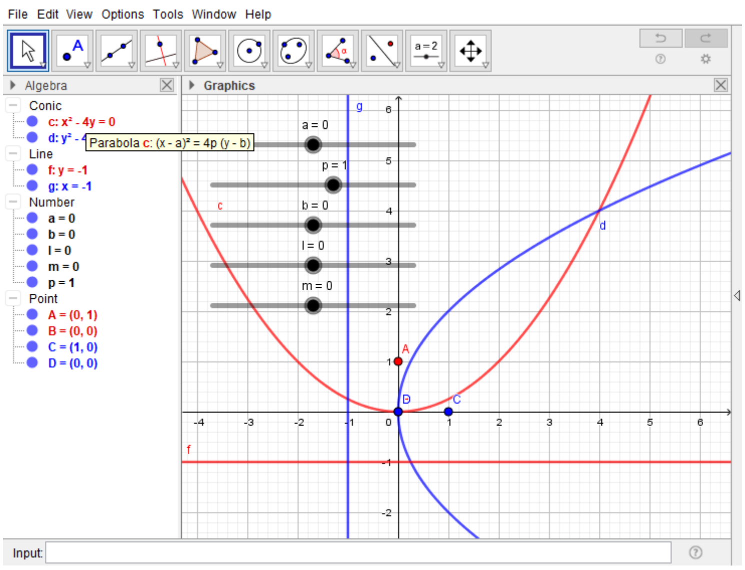
Many bridges are parabolic in shape, such as the **Harbour Bridge** in **Sydney, Australia**.

Note that the standard form is seen when the cursor is placed on red **equation c**. Substituting **a, p** and **b** = 1 (see slider values) gives the standard equation of red **parabola c**, $\mathbf{(x-1)^2=4(y-1)}$. If we solve this equation, we get $x^2-2x+1=4y-4$. Moving terms to opposite sides, we get

$\mathbf{x^2-2x-4y= -5}$ which is the **general equation c** seen in Algebra view.



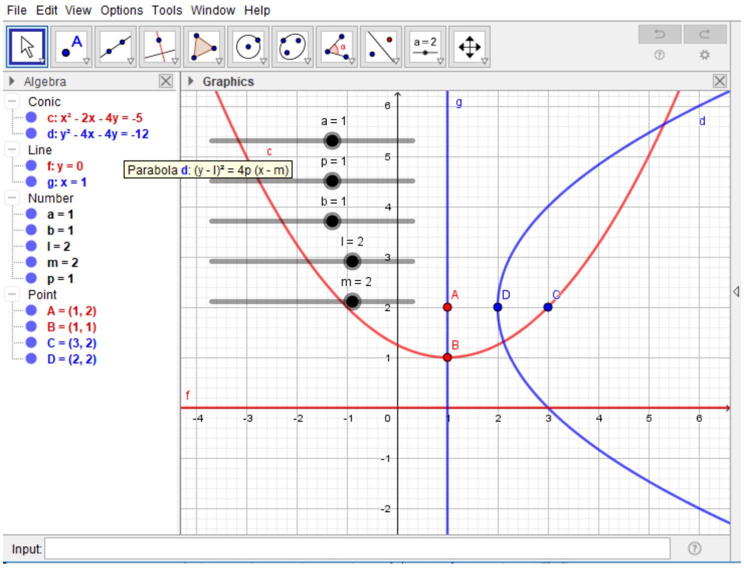
Standard and general equations of **parabola c**; a = p = b = 1 (vertex (1,1))



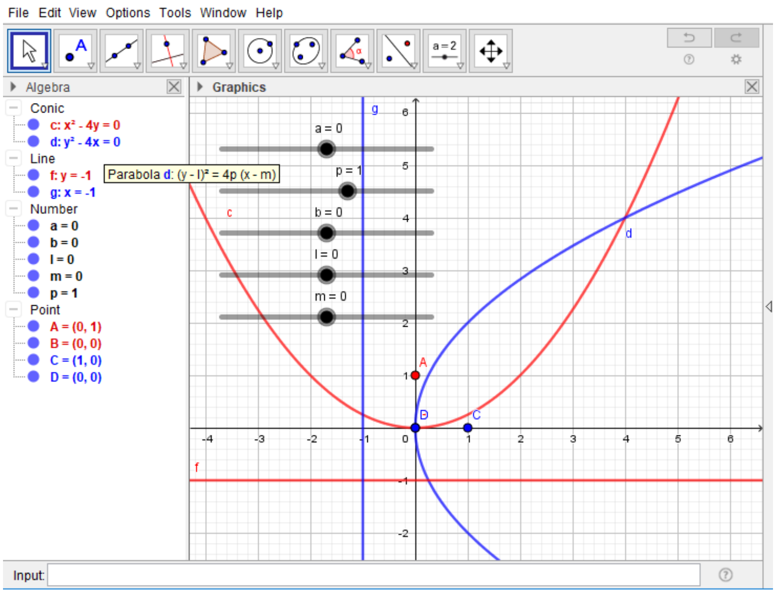
Standard and general equations of **parabola c**; p=1, a = b = 0 (vertex at origin (0,0))

Note that the standard form is seen when the cursor is placed on blue **equation d**. Substituting p=1, l=m=2 (see slider values) gives the standard equation of blue **parabola d**, $(y-2)^2=4(x-2)$. If we solve this equation, we get $y^2-4y+4=4x-8$. Moving terms to opposite sides, we get

$y^2-4x-4y=-12$ which is the general **equation d** seen in Algebra view.



Standard and general equations of **parabola d**; p = 1, l= m=2 (vertex (2,2))



Standard and general equations of **parabola d**; p=1, a = b = 0 (vertex at origin (0,0))