A polynomial is an algebraic expression consisting of terms. It is of the form, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$

For example, $5x^2+6x+2$ has three terms with only variable (x). The coefficients are the real numbers 5, 6 and 2. The degree of this polynomial is the largest exponent, 2 (from the x^2 component). Constant (non-zero), linear, quadratic, cubic and quartic polynomials are of degree 0, 1, 2, 3 and 4, respectively.

Graphs of polynomial functions can be predicted based on their degree, roots and the signs of their first and second derivatives.

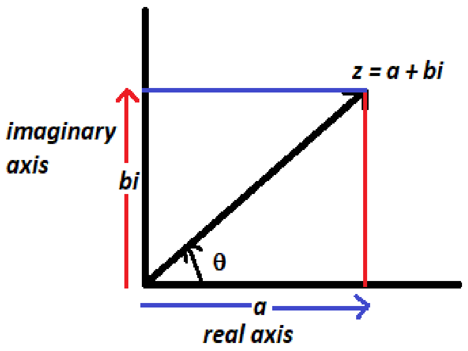
For $f(x) = (x-a) (x-b) (x-c) (x+d)$, its roots are a, b, c and –d. Roots are the solutions to the equations of their functions when the functions are equated to 0.

In the case of f(x), all roots have multiplicity of 1. As 1 is odd, the graph for f(x) cuts the x-axis at (a,0), (b,0), (c,0) and (-d,0). However, for $g(x)=(x-p)^2 (x-q)^3$, there are two repeated roots, x = p with multiplicity 2 and x = q with multiplicity 3. As 2 is even, the graph of g(x) touches the x axis at (p,0). As 3 is off, the graph of g(x) cuts through the x axis at (q,0).

Another way to find the roots of a polynomial is to find the discriminant, which is a polynomial function of its coefficients. For example, for a quadratic polynomial ax^2+bx+c, discriminant Δ is b^2-4ac. As its degree is 2, it has two roots given by $x = (-b \pm \text{sqrt}(b^2-4ac))/2a$. When Δ is 0, its two roots are real and equal. When Δ is positive, it has two distinct real roots. However, when Δ is negative, it has two complex roots.

Graphically, when the roots are real and unequal, the graph is a parabola that intersects the x axis at two points. When the roots are real but equal, the graph is a parabola that touches the x axis at one point. However, when the roots are complex, the graph is a parabola that never intersects the x axis. Its two complex (or imaginary) roots are of the form *a + bi*.

i is an imaginary number equal to the squareroot of -1. In the XY plane, a number *a + bi* corresponds to the point (*a, b*).



In the complex plane, x axis is called the real axis while the y axis is called the imaginary axis. If *z* is a complex number expressed as *a + bi*, then in the complex plane, it is represented by a vector whose real axis co-ordinate is *a* while its imaginary axis co-ordinate is *b*. The length of the vector *z* is *r*, which is also equal to the absolute value of *z*. The angle between this vector at the origin and the real axis in the counter-clockwise direction is the argument *theta* (θ). The polar form of z ($a + bi$) is $r\cos\theta + i\, r\sin\theta$. Thus, the real axis co-ordinate is a or $r\cos\theta$. The imaginary axis co-ordinate is bi or $r\sin\theta$.

Graphically, for a parabola that does not intersect the x axis at all, imagine placing a line parallel to the x axis, passing through its vertex. Now flip the parabola over this line as if it is a mirror and look at the mirror image of this parabola, which now intersects the x axis at two points. Imagine these two points are at the ends of the diameter of a circle. Rotate the circle by 90 degrees. The two complex roots are represented by these two points.

Also helpful in graphing polynomials is knowledge of its inflection points (where the curve changes sign or concavity) and extrema (maxima or minima). There are many resources (textbooks, You Tube videos, teaching materials from colleges and universities) that will help you explore polynomials and their graphs and solutions. Also check out Pascal's triangle which is an easy way to find the coefficients arising from binomial expansion.