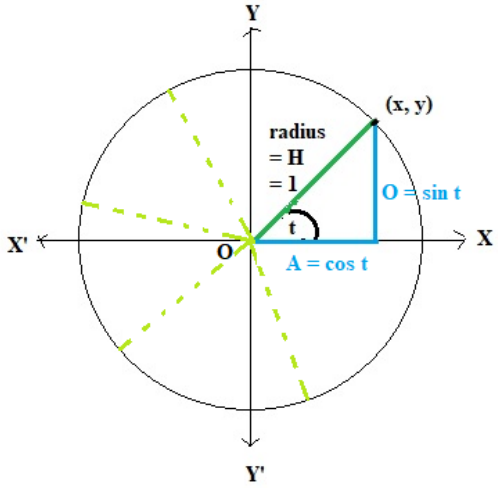
Trigonometry is a branch of mathematics that studies relationships between lengths and angles of triangles. **Trigonometric ratios** provide these relationships. The following table summarizes these trigonometric ratios or **functions**, as they are also known. As these are ratios, they don't have any units.

Trigonometric ratio		
Name	**Ratio**	**Notation**
Sine	Opp/hyp O/H	sin(Θ)
Cosine	Adj/hyp A/H	cos(Θ)
Tangent	Opp/Adj O/A	tan(Θ)
Cosecant	Hyp/opp H/O	cosec(Θ), csc(Θ)
Secant	Hyp/Adj H/A	sec(Θ)
Cotangent	Adj/Opp A/O	cot(Θ)



The same concepts can be visualized in a circle of **radius** of **1**, centered at the **origin**, called a **unit circle**. Imagine a point **(x, y)** on the circle. It starts from the **x axis** and travels in the **counter-clockwise** direction (just to keep the angles positive). Imagine that it is covering **one full rotation**; the central angle (in radians) increases from **0 degrees (0 radians)** to **360 degrees (2*π* radians)** after the rotation when the point returns to its starting place on the x axis. The **radius** joining **(x, y)** to the **origin O** that forms **angle t** with the **x axis** forms the **hypotenuse** of the **right triangle** shown in the figure. The length of the opposite side (blue O) is **y** units whereas the length of the adjacent side A is **x** units.

Remember that **sin t = O/H = y/radius = y/1 = y units**

cos t = A/H = x/radius = x/1 = x units

Inverse trigonometric functions are the inverse functions of trigonometric functions. They are also called **arcus functions** or **antitrigonometric functions**. They are written as **$\cos^{-1}$z (arccos z), $\cot^{-1}$z (arccot z), $\csc^{-1}$z (arccsc z), $\sec^{-1}$z (arcsec z), $\sin^{-1}$z (arcsin z) and $\tan^{-1}$z (arctan z).** Remember **$\cos^{-1}$ z** is not the same as **$(\cos(z))^{-1}$**, which is **1/cos z.**

For **arcsin z = w**, **z = sin w**. This means that w can have multiple values (*e.g.*, sin 60 = sin 120 = 0.866; sin 225 = sin 315 = -0.707). That's why a **principal value** has to be defined and the **domain** has to be restricted. There may be confusion among different texts, but generally, if the **inverse sine** is denoted as $\sin^{-1}$ **z** or **arcsin z**, the principal value is Sin^{-1} **z** or **Arcsin z** (starting with a capital letter).

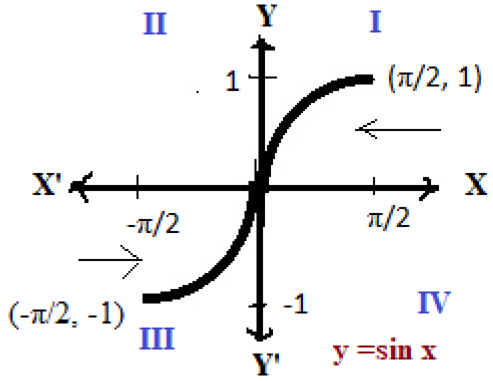
Functions	**Domain**	**Range (Principal value branches)**
$y = \sin^{-1} x$	[-1,1]	-π/2, π/2
$y = \cos^{-1} x$	[-1,1]	0, π
$y = \mathrm{cosec}^{-1} x$	R –(-1,1)*	$-\pi/2 \le y < 0$ or $0 < y \le \pi/2$
$y = \sec^{-1} x$	R –(-1,1)*	$0 \le y < \pi/2$ or $\pi/2 < y \le \pi$
$y = \tan^{-1} x$	R	-π/2, π/2
$y = \cot^{-1} x$	R	0, π

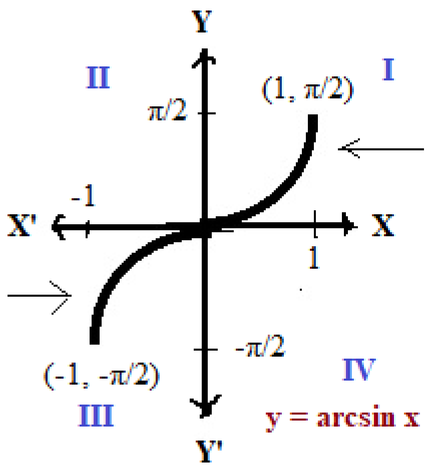
R: all real numbers, *: $x \le -1$ or $x \le 1$

You will get the graph of an inverse trigonometric function when you **interchange x** and **y axes** so that a point **(a, b)** on the graph of the trigonometric function becomes **(b, a)** on the inverse function.

For example, **y= sin x** has the restricted domain **[-π/2, π/2]** and it has three points that we will use: **(π/2, 1), (0, 0)** and **(-π/2, -1)**. Also notice the arrows pointing to the way the curve looks.

To draw the inverse function graph **y =** $\sin^{-1}$ **x**, let's first **interchange** the **x** and **y axes**. So we restrict **x** values between -1 and 1 and the **range** (**y** values) is now -π/2 to π/2. Then, we flip the three points to get three points on the **inverse function graph**: **(1, π/2), (0, 0)** and **(-1, -π/2).** Now we even change the **curvature**, what was close to the **y axis** in the **first quadrant** is now close to the **x axis**, and *vice versa.*

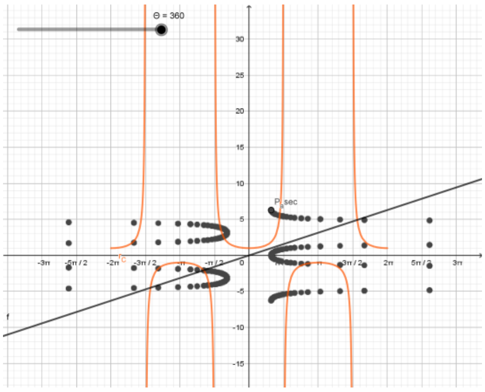




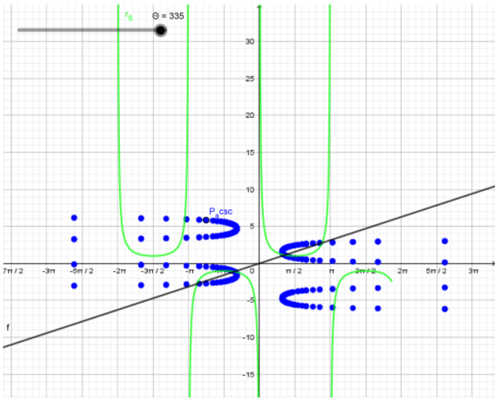
<u>**Hints for the assignment in the tutorial:**</u>

1) For the secant, cosecant and cotangent functions, use **r_C, r_S** and **r_T**, respectively, to define the functions. **r** stands for **reciprocal function**. Remember that **secant** is reciprocal of **cosine** and so on.
2) For secant and cosecant, use the **domain** from -2π to α (=2π).
3) For cotangent, use the domain from –∞ to ∞.
4) For inverse secant, cosecant and cotangent functions, use P_asec, P_acosec and P_acot to create points that will trace these inverse functions.

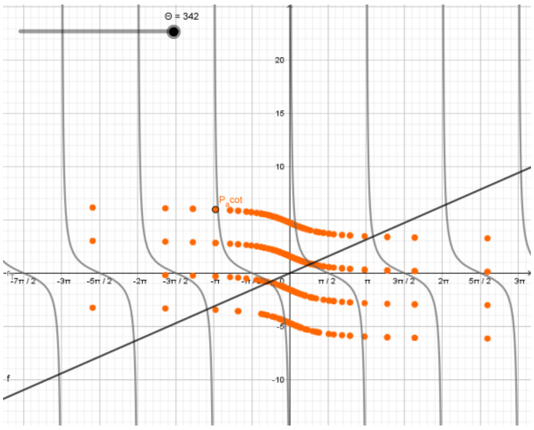
5) For example, for secant, you would type, in the input bar, r_C:=Function[sec(x), -2π, α] and for arcsecant, you would type P_sec=(sec(α),α).
6) Inverse functions are usually drawn after restricting the domains, and even the ranges, of the corresponding trigonometric functions. However, for now, you can focus on just the points tracing the inverse function graphs.
7) Also draw y=x to check reflection of the trigonometric functions and their inverse functions.
8) Your completed graphs should look like this. The graphs stretched vertically are all trigonometric functions. The graphs that are squeezed around the x axis are all inverse trigonometric functions.



Secant, arcsecant



cosecant, arccosecant



Cotangent, arccotangent

9) To graph the restricted inverse trigonometric functions, we would need to treat them as discontinuous functions and graph the different pieces separately. We will look at discontinuous functions later.